

Multi-Regional Growth, Agglomeration and Land Values in a Generalized Heckscher-Ohlin Trade Model

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Abstract

This paper builds a general equilibrium interregional trade model with any number of regional economies by unifying the Solow-Uzawa growth and Heckscher-Ohlin (H-O) trade theories. We study the roles of population, land, preferences and technological differences between regions in determining dynamics of national wealth, patterns of interregional trade, economic structures, and income and wealth distributions between different regional households in a general equilibrium dynamic framework. The paper deals with endogenous wealth accumulation, land values, residential land-use, and labor and capital distribution between sectors and between regions under perfectly competitive markets and free trade. Each region has three sectors, producing one national homogenous tradable capital good, specifying in producing one tradable commodity, and supplying non-tradable goods and services. We simulate the model to show existence of an equilibrium point and movement of the interregional economy. We examine the effects on the equilibrium point in exogenous changes of the land, the population, a region's amenities, and the preferences for a regional good, saving, housing, and residential density.

Keywords: interregional economies; H-O model; Solow-Uzawa model; land value; neoclassical growth theory; inequality in income and wealth.

JEL classification: O41, F11, D31.

Introduction

One of the most important issues in contemporary economies is spatial agglomeration. People and production activities tend to be concentrated in a few regions. In association with spatial agglomeration, another important issue is related to how income and wealth distributions between regions and between different groups in different regions evolve over time. Most of the literature in interregional economics has failed to provide comprehensive insights into this kind of problem. A main reason for dealing with growth and spatial agglomeration is that in order to properly deal with this kind of problems, one often has to solve complicated nonlinear dynamic equations. Without a computer it is almost impossible to study interregional trade and growth. The purpose of this study is to examine these issues in a dynamic equilibrium framework by integrating

the traditional Solow-Uzawa growth and Heckscher-Ohlin (H-O) trade theories. With simulation, this study examines interregional interactions between national economic growth, regional economic growth, economic structures, and income and wealth between different regions by unifying the Solow-Uzawa growth and Heckscher-Ohlin trade theories within an integrated framework with Zhang's approach to households' behavior (Zhang, 1993a).

With regard to interregional trade we follow the Heckscher-Ohlin (H-O) model (Heckscher, 1919, Ohlin, 1933, and Heckscher and Ohlin, 1991). Many efforts have been done to introduce dynamic elements to H-O models (e.g., Ventura, 1997, Bajona and Kehoe, 2010). A standard H-O model is for a two-countries global economy, each country having access to the same technology for producing two goods using two fixed factors (labor and capital) under conditions of perfect competition and constant returns to scale. International trade for factors of production is not allowed. Factors of production are mobile between sectors within a country. No international borrowing and lending is allowed. According to Chen (1992: 923-4),

“It appears to be the general consensus in this body of literature that the main determinant of long-run comparative advantage is the countries' savings rates. The question of what has caused the difference in savings rates among countries, however, is rarely explicitly discussed in the literature. The models that do endogenize savings rates (e.g., Stiglitz 1970) attribute the difference in savings rates and hence long-run comparative advantage to a difference in preferences; in particular, a difference in agents' time discount factors among countries. Yet explaining trade in terms of differences in preferences is no longer in the spirit of the Heckscher-Ohlin model in which trade arises because of differences in relative factor proportions.”

This study allows factor mobility and deals with a global economy with any number of regional economies. It should be noted that our model is Ricardian as it postulates cross-region differences in technology and labor productivity.

The model is based on the neoclassical growth theory in the sense that it treats endogenous wealth as a main engine of global economic growth and the economic structure and production technologies are neoclassical. With product, labor and capital mobility this study follows the neoclassical growth theory, assuming perfect competition and full employment of input factors (e.g., Burmeister and Dobell, 1970;

Azariadis, 1993; Barro and Sala-i-Martin, 1995). The model in this study is also based on the dynamic models with endogenous capital by Oniki and Uzawa and others (e.g., Oniki and Uzawa, 1965; Deardorff and Hanson, 1978; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). The Oniki-Uzawa model is constructed for the two-country with two goods with fixed saving rates. The Oniki-Uzawa model is often used as a start point for analyzing interdependence between trade patterns and economic growth. In most models of this type goods and services are classified into capital goods and consumer goods (Stulz, 1987; Stockman and Dellas, 1989; Edwards, 1989; Mendoza, 1995; Pesenti and van Wincoop, 2002; Rogoff, 2002; Zhao *et al.*, 2014). Backus and Smith (1993:1) emphasized the significance of tradable and non-tradable goods in trade theory as follows: “The mechanism is fairly simple. Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and nontraded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries.”

We introduce distinct production sectors in order to examine trade patterns with taking account of these issues.

Regional agglomeration has been observed not only in developed economies, but also increasingly more in many underdeveloped economies. Bairoch (1993) argues that rapid disparities between rich and poor regions are modern phenomena. Kuznets (1966) attributed this phenomenon to industrialization. The earlier development theories by Myrdal (1957) and Hirschman (1958) argue for dynamic interactions between industrial growth and the geographical concentration of industry: industrialization attracts resources to a given location and the resulting agglomeration stimulates growth. There are numerous studies on various aspects of spatial economic developments. Our approach differs from the so-called new economic geography (e.g., Krugman, 1991, 1993; Ottaviano *et al.*, 2002; Forslid and Ottaviano, 2003; Pflüger, 2004; Charlot, *et al.* 2006; Picard and Tabuchi, 2010). In almost all the dynamic models of the new economic geography, physical capital is completely neglected and regional amenities do not play a significant role in determining land rent and population mobility. Although this approach is claimed to have “enabled researchers to gain further insights into the space economy and its transition” (Tabuchi, 2014: 50), it is difficult to imagine any modern economy whose dynamics can be

properly modelled neither with wealth accumulation and nor amenity. Indeed, in almost all of these regional dynamic models there is no consideration of amenity and land value determination. Amenities have increasingly caused attention from spatial scientists (Glaeser *et al.*, 2001; Partridge *et al.*, 2008a, 2008b; Chen *et al.*, 2013). There is a large body of literature on amenities and spatial economics, for instance, equilibrium ideas by Graves (1979) and Roback (1982), turnaround migration theory by Brown *et al.* (1997), life cycle studies by Clark and Hunter (1992), research on rural development by Deller *et al.* (2001). Zhang (1993b) first introduced spatial amenity into utility in a general equilibrium framework.

A main concern of this study is dynamic interdependence between economic growth and land value change. Although issues related to land value change and economic structural change are significant, economics still needs an analytical framework for analyzing these issues. The literature on house and land prices has been increasingly expanding in recent years as surveyed by Cho (1996: 145), “During the past decade, the number of studies on intertemporal changes in house prices has increased rapidly because of wider availability of extensive micro-level data sets, improvements in modeling techniques, and expanded business applications.” (e.g., Bryan and Colwell, 1982; Case and Quigley, 1991; Chinloy, 1992; Clapp and Giaccotto, 1994; Calhoun, et al. 1995; Quigley, 1995; Capozza and Seguin, 1996; Alpanda, 2012; Du and Peiser, 2014; Kok *et al.* 2014). Most of these studies are empirical. There are only a few formal growth models with endogenous land values. This paper is based on the three models proposed by Zhang. The economic structure is based on Zhang (2015), which proposes a generalized multinational H-O model with three sectors in each country. The regional amenity and population distribution mechanism are based on Zhang (2016). The determinants of land values are influenced by Zhang (2014). The rest of this paper is arranged as follows. Section 2 develops the interregional growth model of wealth and income distribution and regional amenity. Section 3 examines dynamic properties of the model and simulates the model. Section 4 studies transitory effects and long-run equilibrium shifts when some parameters are exogenously changed. Section 5 gives conclusions to the study. The results in Section 3 are confirmed in the Appendix.

The Model

As mentioned before, the model in this paper based on the three models by Zhang. The production within each region is based on the neoclassical trade theory with capital accumulation. The trade aspects are based on some typical dynamic H-O models and the neoclassical trade growth theory. Different from the standard H-O model with two countries, the economy consists of any number of regional economies, indexed by $j = 1, \dots, J$. Also different from the standard H-O model with only two goods, this study assumes that each region produces three goods. Each region produces a homogeneous capital good which can be used as capital and consumption. This sector is called industrial sector. The industrial sector is similar to the homogenous sector in the traditional neoclassical trade model (e.g., Ikeda, and Ono, 1992). Capital goods and people are freely mobile between regions. Each region produces services which are not interregionally tradable and are consumed only by domestic households. Each region specifies in producing a good called national commodity which is interregionally tradable. National commodities are pure consumption goods. Production sectors employ two inputs - capital and labor. All markets are perfectly competitive. Factors are fully employed at any point in time. Different from the traditional H-O model, labor is interregional mobile. Only households undertake saving. All earnings that firms make are distributed as payments to factors of production.

We assume perfect competition in all the markets both within region and between regions. Commodities are traded without any barriers. We neglect transport costs. We measure prices in terms of capital good and the price of capital good is unity. We use $w_j(t)$ and $r_j(t)$ to stand for, respectively, wage and interest rates in the j th region. The interest rate is equalized throughout the national economy, i.e., $r(t) = r_j(t)$. We consider that the national population N is fixed and homogenous. Workers are free to choose their work place and residential location. Workers work and reside in the same region. Each region has fixed land which is used only for residential purposes. Land is owned privately and people from any region can own land in any region. The assumption of zero transportation cost of commodities implies price equality for tradable commodities between regions. We assume that regions are different in productivities and living conditions. We use amenity to measure living conditions. As

amenity and land are immobile, wage rates, land values and land rents may vary between regions.

Let $N_j(t)$ stand for region j 's flow of labor services for production. The labor force is distributed between the three sectors. We use subscripts, i , s , and m to denote industrial sector, service sector, and region j 's global commodity sector, respectively. We use (j, q) to index sector q , $q = i, s, m$, in region j . We use $p_{jq}(t)$ to represent the price of product (j, q) . Labor and capital are paid at their marginal products. Firms have zero profits. We assume different capital depreciation rates between regions. The capital employed by region j depreciates at a constant exponential rate δ_{jk} . We use $N_{jq}(t)$ and $K_{jq}(t)$ to stand for the labor force and capital stocks employed by sector q in region j . Let $F_{jq}(t)$ stand for the output level of sector q in region j . Let $K_j(t)$ stand for the capital stocks employed by region j 's production sectors.

The production functions

The production function of section q in region j is specified as

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \quad A_{jq}, \alpha_{jq}, \beta_{jq} > 0, \quad \alpha_{jq} + \beta_{jq} = 1, \quad (1)$$

where A_{jq} , α_{jq} , and β_{jq} are non-negative constants. It should be noted that the traditional H-O model assumes variations in capital and labor endowments with the identical technology between countries. In this study we assume not only differences in capital and labor endowments like in the H-O model but also differences in technologies between regions.

The marginal conditions for the production sectors

The rate of interest, wage rate, and prices are given to individual firms and are decided in markets. Accordingly, these variables are given for each individual firm at each point in time. Sector (j, q) maximizes its following profit by choosing $K_{jq}(t)$ and $N_{jq}(t)$

$$p_{jq}(t)F_{jq}(t) - (r(t) + \delta_{jk})K_{jq}(t) - w_j(t)N_{jq}(t),$$

where $p_{ji}(t) = 1$. The marginal conditions imply

$$r(t) + \delta_{jk} = \frac{\alpha_{jq} p_{jq}(t) F_{jq}(t)}{K_{jq}(t)}, \quad w_j(t) = \frac{\beta_{jq} p_{jq}(t) F_{jq}(t)}{N_{jq}(t)}. \quad (2)$$

Choice between (physical) wealth and land

Land may be owned by different agents under various institutions. This study assumes that land is owned privately by households. For simplicity, we assume that land can be sold and bought in free markets without any friction and transaction costs. Land use will not waste land and land cannot regenerate itself. Households own land and physical wealth. Region j 's land is L_j and the land rent is $R_j(t)$. We use $p_{jL}(t)$ to denote the price of land in region j . Consider now an investor with one unity of money. He can either invest in capital good thereby earning a profit equal to the net own-rate of return $r(t)$ or invest in land thereby earning a profit equal to the net own-rate of return $R_j(t)/p_{jL}(t)$. As we assume capital and land markets to be at competitive equilibrium at any point in time, two options must yield equal returns, i.e.

$$\frac{R_j(t)}{p_{jL}(t)} = r(t). \quad (3)$$

This equation enables us to determine choice between owning land and wealth. Obviously this assumption is made under many strict conditions. For instance, we neglect any transaction costs and any time needed for buying and selling. Expectations on land are complicated. Equation (3) also implies perfect information and rational expectation.

The current income and disposable income

For simplicity, we use lot size to stand for housing. As argued, for instance, by Davis and Heathcote (2007), most of the fluctuations in house prices are driven by land price rather than by the cost of structures. This study applies the utility function proposed by Zhang (1993a) to describe behavior of households. We denote respectively physical wealth $\bar{k}_j(t)$ and region n 's

land in region $\bar{l}_{jn}(t)$ owned by the representative household in region j . The total value of wealth owned by the household $a_j(t)$ is the sum of the two assets

$$a_j(t) = \bar{k}_j(t) + \sum_{n=1}^J p_{Ln}(t) \bar{l}_{jn}(t). \quad (4)$$

Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, the wage payment $w_j(t)$, and the land revenue $\sum_{n=1}^J R_n(t) \bar{l}_{jn}(t)$ is

$$y_j(t) = r(t)\bar{k}_j(t) + w_j(t) + \sum_{n=1}^J R_n(t) \bar{l}_{jn}(t). \quad (5)$$

We call $y_j(t)$ the current income as it is composed of a consumer's wage income and current interest income of wealth. The representative household's disposable income is

$$\hat{y}_j(t) = y_j(t) + a_j(t). \quad (6)$$

The disposable income is distributed between consumption and saving.

The budget, utility, and optimal behavior

The household spends the disposable income on housing $l_j(t)$, consumption of services $c_{js}(t)$, consumption of capital good $c_{ji}(t)$, regional goods $c_{jn}(t)$, and saving $s_j(t)$. The budget constraint is

$$R_j(t)l_j(t) + p_{js}(t)c_{js}(t) + c_{ji}(t) + \sum_{n=1}^J p_{nm}(t)c_{jn} + s_j(t) = \hat{y}_j(t). \quad (7)$$

This equation implies that the household's disposable income is distributed between the consumption and saving. The utility function,

$U_j(t)$, of the household is dependent on $l_j(t)$, $c_{js}(t)$, $c_{ji}(t)$, $c_{jn}(t)$, and $s_j(t)$ as follows

$$U_j(t) = \theta_j(t) l_j^{\eta_0}(t) c_{ji}^{\xi_0}(t) c_{js}^{\chi_0}(t) s_j^{\lambda_0}(t) \prod_{n=1}^J c_{jn}^{\gamma_{0n}}(t), \quad \eta_0, \xi_0, \chi_0, \gamma_{0n}, \lambda_0 > 0, \quad (8)$$

where η_0 is called household's propensity to use lot size, ξ_0 propensity to consume capital good, χ_0 propensity to consume service, γ_{0n} propensity to consume regional commodity n , and λ_0 propensity to own wealth. In (8) $\theta_j(t)$ is called region j 's amenity level. Amenities are affected by environment, infrastructures, regional cultures and climates (e.g., Kanemoto, 1980; Diamond and Tolley, 1981; Blomquist, *et al.* 1988; Cho *et al.*, 2008; Landry and Chakraborty, 2009; Lanz and Provins, 2013; Leknes, 2015, and Brinkman, 2016). This study treats amenities as endogenous variables. It is assumed that amenity is affected by level of population. We specify values of θ_j as follows

$$\theta_j(t) = \bar{\theta}_j N_j^d(t), \quad j = 1, \dots, J, \quad (9)$$

where $\bar{\theta}_j (> 0)$, $\omega (< 0)$, and d are parameters, and $N_j(t)$ is region j 's population. We don't specify signs of d as population may have either positive or negative effects on regional attractiveness. As Chen *et al.* (2013: 269) observe: "The presence of both positive and negative population externalities suggests that the steady state (or competitive) pattern may differ from an optimal pattern in which all the external benefits and costs of households' migration decisions are internalized."

Maximizing (8) subject to the budget constraint (7) implies

$$\begin{aligned} l_j(t) &= \frac{\eta \hat{y}_j(t)}{R_j(t)}, \quad c_{ji}(t) = \xi \hat{y}_j(t), \quad c_{js}(t) = \frac{\chi \hat{y}_j(t)}{p_{js}(t)}, \\ c_{jn}(t) &= \frac{\gamma_n \hat{y}_j(t)}{p_{nm}(t)}, \quad s_j(t) = \lambda \hat{y}_j(t), \end{aligned} \quad (10)$$

where

$$\eta \equiv \rho \eta_0, \quad \chi \equiv \rho \chi_0, \quad \xi \equiv \rho \xi_0, \quad \gamma_n \equiv \rho \gamma_{0n}, \quad \lambda \equiv \rho \lambda_0,$$

$$\rho \equiv \frac{1}{\eta_0 + \chi_0 + \xi_0 + \lambda_0 + \sum_{n=1}^J \gamma_{0n}}.$$

Wealth accumulation

According to the definition of $s_j(t)$, the change in wealth of the representative household from group j is

$$\dot{a}_j(t) = s_j(t) - a_j(t). \quad (11)$$

This equation simply implies that the change in wealth is the saving minus dissaving.

Equalization of utility levels between regions

We assume that households are freely mobile between the regions. The utility levels of homogenous population between regions are assumed to be equal at any point in time. That is

$$U_j(t) = U_q(t), \quad j, q = 1, \dots, J. \quad (12)$$

Here, we omit possible costs for migration. Changing houses or moving to another region will cost. Taking account of such changes in the model makes it difficult to analyze the model. It should be noted that wage equalization between regions is often used as the equilibrium mechanism of population mobility over space. In our approach instead of wage equalization it is assumed that households obtain the same level of utility in different regions as the equilibrium mechanism of population distribution between regions.

Factor marketing clearing conditions

In each region the capital stock is fully employed by the three sectors. That is

$$K_{ji}(t) + K_{jm}(t) + K_{js}(t) = K_j(t). \quad (13)$$

In each region the labor force is fully employed by the three sectors

$$N_{ji}(t) + N_{jm}(t) + N_{js}(t) = N_j(t). \quad (14)$$

Market clearing for services

The demand and supply of services balance in each regional market

$$c_{js}(t)N_j(t) = F_{js}(t). \quad (15)$$

Market clearing in global commodity markets

The demand and supply of national goods balance in national markets

$$\sum_{j=1}^J c_{jn}(t)N_j(t) = F_n(t). \quad (16)$$

Market clearing in capital markets

The change in the total capital stock is the national capital production minus the total depreciation of capital stocks and national consumption. That is

$$\dot{K}(t) = \sum_{j=1}^J F_{ji}(t) - \sum_{j=1}^J \delta_{kj} K_j(t) - \sum_{j=1}^J c_{ji}(t)N_j(t). \quad (17)$$

The balance of the capital stock and total physical wealth

The physical wealth owned by the national population is equal to the national capital stock

$$\sum_{j=1}^J \bar{k}_j(t)N_j(t) = \sum_{j=1}^J K_j(t) = K(t). \quad (18)$$

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The land owned by the population

The land owned by the population is equal to the regional land

$$\sum_{n=1}^J \bar{l}_{nj}(t) N_n(t) = L_j. \quad (19)$$

Land resource being utilized fully

We assume that the regional land is fully used for residential use

$$l_j(t) N_j(t) = L_j, \quad j = 1, \dots, J. \quad (20)$$

We constructed the multi-regional growth model. The growth mechanism of the model is neoclassical in the sense that endogenous capital accumulation is the key determinant of economic growth. Nevertheless, it differs from traditional neoclassical growth theory in the sense that there are endogenous rational mechanisms for determination of regional capital and labor distribution, land use distribution, and land value in the national economy.

The Dynamics and Equilibrium

The previous section developed the national growth model with any number of regional economies. As the system is nonlinear and is of high dimension, it is difficult to obtain analytical properties of the system. Before calibrating the model, we first introduce a variable

$$z_1 \equiv \frac{r + \delta_{1k}}{w_1}.$$

The following lemma shows how to follow the dynamics of global economic growth with initial conditions.

Lemma

We describe the movement of J -region economy by the following $J + 1$ differential equations with $J + 1$ variables $z_1(t)$ and (a_j)

$$\begin{aligned}\dot{z}_1(t) &= \tilde{\Lambda}(z_1(t), (a_j(t))), \\ \dot{a}_j(t) &= \tilde{\Lambda}_j(z_1(t), (a_j(t))), \quad j = 1, \dots, J,\end{aligned}\tag{21}$$

where $\tilde{\Lambda}_1(t)$ and $\tilde{\Lambda}_j(t)$ are functions of $z_1(t)$ and $(\dot{a}_j(t))$, given in the Appendix. We find the values of the rest variables as functions of $z_1(t)$ and $(\dot{a}_j(t))$ at any point in time as follows: $r(t)$ by (A2) $\rightarrow w_j(t)$ by (A2) $\rightarrow z_j(t)$ by (A3) $\rightarrow p_{jq}(t)$ by (A4) $\rightarrow \hat{y}_j(t)$ by (A7) $\rightarrow c_{ji}(t)$, $c_{js}(t)$, $c_{jn}(t)$, and $s_j(t)$ by (7) $\rightarrow N_1(t)$ by (A20) $\rightarrow N_j(t)$ by (A19) $\rightarrow K(t)$ by (A16) $\rightarrow K_j(t)$ by (A15) $\rightarrow N_{jm}(t)$ by (A14) $\rightarrow N_{ji}(t)$ by (A12) $\rightarrow N_{js}(t)$ by (A11) $\rightarrow K_{jq}(t)$ by (A1) $\rightarrow F_{jq}(t)$ by (2) $\rightarrow l_j(t)$ by (20) $\rightarrow R_j(t)$ by (7).

The lemma confirms that we have a set of nonlinear differential equations from which we can explicitly determine the motion of the $J + 1$ variables. The computational procedure given in the lemma enables us to plot the motion of the economic system once the initial conditions are. Following the procedure with computer, we can illustrate the motion of the system. We consider a national economy with 3 regions. For simulation, we specify values of the parameters as follows

$$\lambda_0 = 0.7, \quad \xi_0 = 0.05, \quad \chi_0 = 0.07, \quad \eta_0 = 0.04, \quad d = -0.06, \quad \gamma_{01} = 0.01, \\ \bar{\theta}_1 = 4.1, \quad \bar{\theta}_2 = 4.2, \quad \bar{\theta}_3 = 4.3,$$

$$\begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{3i} \\ \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{3i} \end{pmatrix} = \begin{pmatrix} 1.4 \\ 1.3 \\ 1.2 \\ 0.32 \\ 0.31 \\ 0.3 \end{pmatrix}, \quad \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \\ \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \\ 0.3 \\ 0.32 \\ 0.32 \end{pmatrix}, \quad \begin{pmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \\ \alpha_{1m} \\ \alpha_{2m} \\ \alpha_{3m} \end{pmatrix} = \begin{pmatrix} 1.3 \\ 1.2 \\ 1.1 \\ 0.34 \\ 0.3 \\ 0.31 \end{pmatrix}, \quad \begin{pmatrix} L_1 \\ L_2 \\ L_3 \\ \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 7 \\ 0.05 \\ 0.055 \\ 0.06 \end{pmatrix}.$$

The national population is 100. The total factor productivities are different between the regional economies. The total factor productivity of region 1's sector is higher than that of region 2's corresponding sector and region

2's is higher than that of region 3's. We specify the initial conditions as follows:

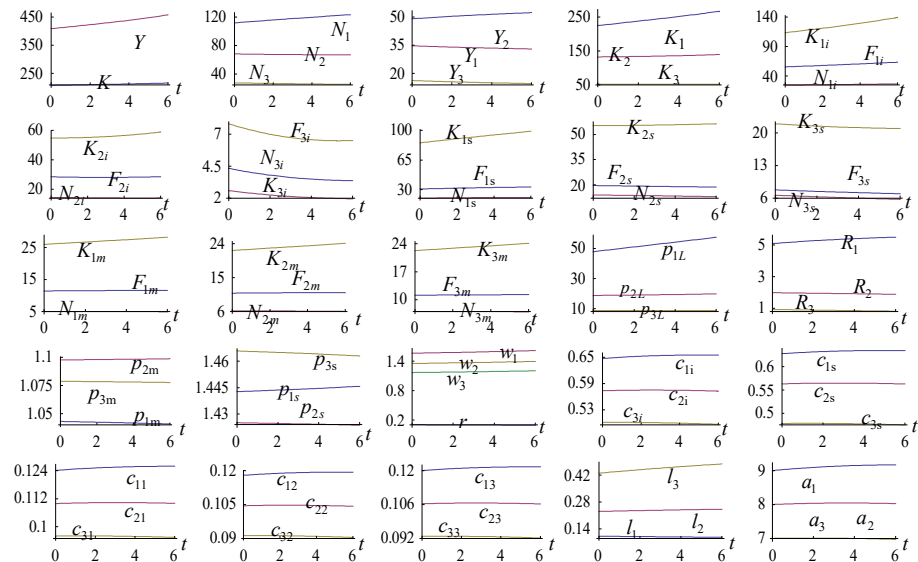
$$z_1(0) = 0.1, \quad a_1(0) = 9, \quad a_2(0) = 8, \quad a_3(0) = 7.$$

The motion of the system is given in Figure 1. In the figure the regional incomes and the national income are defined as follows

$$Y_j(t) \equiv F_{ji}(t) + p_{js}(t)F_{js}(t) + p_j(t)F_{jj}(t), \quad Y(t) \equiv Y_1(t) + Y_2(t) + Y_3(t).$$

The national income slightly rises and wealth rises. Region 1's population and capital stocks are increased and the other two regions' populations and capital stocks are slightly reduced. Some of the sectors produce less and employ less capital stocks and some other sectors produce more and employ more capital stocks. Region 1's price of service rises and the other two regions' prices fall. The prices of the regional goods change slightly. The changes in the other variables are illustrated in Figure 1.

Fig. 1.
The motion of the
global economy



From Figure 1 we observe that the system does not become stationary in the long term. We identified an unstable equilibrium point, the point is not stable. Following the procedure in the lemma, we calculate the equilibrium values of the variables as follows

$$Y = 209, \quad K = 410, \quad r = 0.107,$$

$$\begin{aligned} \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} &= \begin{pmatrix} 124 \\ 63 \\ 23 \end{pmatrix}, \quad \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 54.4 \\ 31.9 \\ 13.7 \end{pmatrix}, \quad \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 246.2 \\ 120.3 \\ 43.5 \end{pmatrix}, \quad \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 59.3 \\ 23.8 \\ 0.99 \end{pmatrix}, \\ \begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} &= \begin{pmatrix} 35.8 \\ 18.4 \\ 6.6 \end{pmatrix}, \quad \begin{pmatrix} F_{1m} \\ F_{2m} \\ F_{3m} \end{pmatrix} = \begin{pmatrix} 12 \\ 11.4 \\ 11.6 \end{pmatrix}, \quad \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 25.9 \\ 12.1 \\ 0.59 \end{pmatrix}, \quad \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 23.2 \\ 13.2 \\ 5.7 \end{pmatrix}, \\ \begin{pmatrix} N_{1m} \\ N_{2m} \\ N_{3m} \end{pmatrix} &= \begin{pmatrix} 5.3 \\ 6.5 \\ 7.4 \end{pmatrix}, \quad \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 120.6 \\ 45.5 \\ 1.77 \end{pmatrix}, \quad \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 98.5 \\ 51.7 \\ 18.6 \end{pmatrix}, \quad \begin{pmatrix} K_{1m} \\ K_{2m} \\ K_{3m} \end{pmatrix} = \begin{pmatrix} 27 \\ 23 \\ 23 \end{pmatrix}, \\ \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} &= \begin{pmatrix} 1.56 \\ 1.35 \\ 1.17 \end{pmatrix}, \quad \begin{pmatrix} p_{1L} \\ p_{2L} \\ p_{3L} \end{pmatrix} = \begin{pmatrix} 55 \\ 17.5 \\ 7.4 \end{pmatrix}, \quad \begin{pmatrix} p_{1s} \\ p_{2s} \\ p_{3s} \end{pmatrix} = \begin{pmatrix} 1.44 \\ 1.43 \\ 1.47 \end{pmatrix}, \quad \begin{pmatrix} p_{1m} \\ p_{2m} \\ p_{3m} \end{pmatrix} = \begin{pmatrix} 1.04 \\ 1.1 \\ 1.08 \end{pmatrix}, \\ \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} &= \begin{pmatrix} 5.9 \\ 1.8 \\ 0.8 \end{pmatrix}, \quad \begin{pmatrix} c_{1i} \\ c_{2i} \\ c_{3i} \end{pmatrix} = \begin{pmatrix} 0.68 \\ 0.59 \\ 0.51 \end{pmatrix}, \quad \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 0.66 \\ 0.58 \\ 0.49 \end{pmatrix}, \quad \begin{pmatrix} c_{11} \\ c_{21} \\ c_{31} \end{pmatrix} = \begin{pmatrix} 0.13 \\ 0.11 \\ 0.1 \end{pmatrix}, \\ \begin{pmatrix} c_{12} \\ c_{22} \\ c_{32} \end{pmatrix} &= \begin{pmatrix} 0.12 \\ 0.11 \\ 0.09 \end{pmatrix}, \quad \begin{pmatrix} c_{13} \\ c_{23} \\ c_{33} \end{pmatrix} = \begin{pmatrix} 0.13 \\ 0.11 \\ 0.09 \end{pmatrix}, \quad \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 0.09 \\ 0.25 \\ 0.51 \end{pmatrix}, \quad \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 9.49 \\ 8.22 \\ 7.11 \end{pmatrix}. \end{aligned} \quad (22)$$

It is straightforward to calculate the four eigenvalues as follows $\{0.185, -0.188, -0.129, -0.129\}$.

Dynamic Comparative Analysis

The previous section simulated the motion of the dynamic system. This section carries out comparative analysis. As we can follow the motion of the economy, it is straightforward to provide transitory and long-term effects of changes in any parameter on the economy. We now study how a change in some exogenous condition affects the regional economies and national economy. To plot motion of variables, a variable $\bar{\Delta}x(t)$ is used to

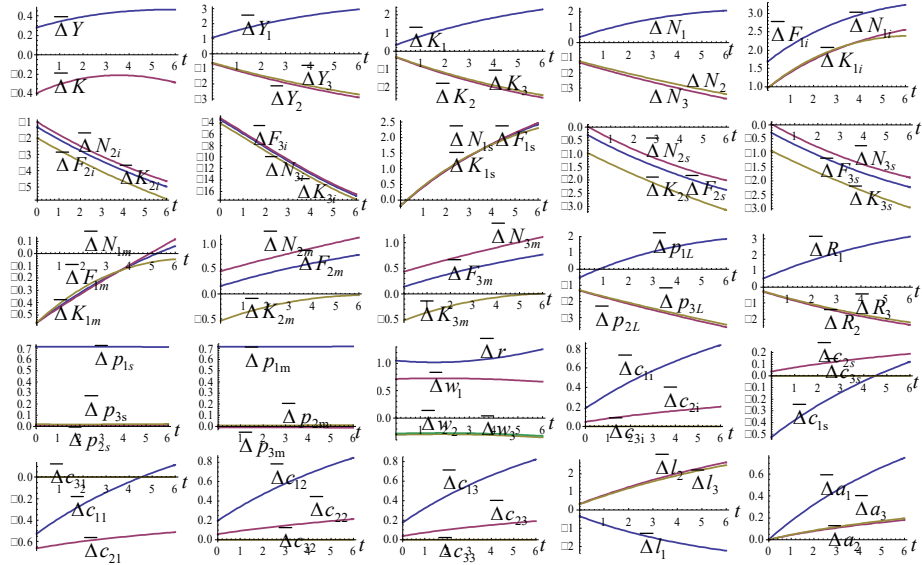
represent the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

A rise in the total factor productivity of a region's industrial sector

We now study effects of an improvement in the total factor productivity of region 1's industrial sector. We allow the total factor productivity to be changed as follows $A_{i1} : 1.4 \Rightarrow 1.41$. The results are plotted in Figure 2. As the system is unstable, we simulate the model only in a short-run period. We see that the change in region 1's productivity increases the national income and region 1's income and reduces the national capital and the other two regions' incomes. More people move from the other two regions to region 1. The production scales of region 1's three sectors are expanded while the other two regions' production sectors reduce the output levels. The land price and rent are increased in region 1 and are reduced in the other two regions. In micro level region 1's typical household has more wealth, consume more, and uses smaller lot size, while the other two regions' households have less wealth, consume less, and use larger lot size.

Since the system is unstable, in the rest of this study we simulate the impact on the equilibrium point in order to see how each parameter may affect the economic structure. The impact on the equilibrium point is listed in (23). Comparing the impact on the equilibrium point in (23) and the transitory process in the figure, we see that the signs of some equilibrium variables are not the same as the signs of the variables in the transitory process. For instance, the equilibrium value of national capital is increased, while the short-run change direction of the variable falls.

Fig. 2.
A rise in the
total factor
productivity of
a region's
industrial sector



$$\bar{\Delta Y} = 1.01, \quad \bar{\Delta K} = 1.07, \quad \bar{\Delta r} = 0.03,$$

$$\begin{pmatrix} \bar{\Delta Y}_1 \\ \bar{\Delta Y}_2 \\ \bar{\Delta Y}_3 \end{pmatrix} = \begin{pmatrix} 4.87 \\ -4.57 \\ -4.57 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta N}_1 \\ \bar{\Delta N}_2 \\ \bar{\Delta N}_3 \end{pmatrix} = \begin{pmatrix} 3.8 \\ -4.54 \\ -4.54 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta K}_1 \\ \bar{\Delta K}_2 \\ \bar{\Delta K}_3 \end{pmatrix} = \begin{pmatrix} 4.82 \\ -4.62 \\ -4.42 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta F}_{1i} \\ \bar{\Delta F}_{2i} \\ \bar{\Delta F}_{3i} \end{pmatrix} = \begin{pmatrix} 5.65 \\ -7.55 \\ -74.36 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta F}_{1s} \\ \bar{\Delta F}_{2s} \\ \bar{\Delta F}_{3s} \end{pmatrix} = \begin{pmatrix} 4.14 \\ -4.53 \\ -4.52 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta F}_{1m} \\ \bar{\Delta F}_{2m} \\ \bar{\Delta F}_{3m} \end{pmatrix} = \begin{pmatrix} 0.34 \\ 1.03 \\ 1.03 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta N}_{1i} \\ \bar{\Delta N}_{2i} \\ \bar{\Delta N}_{3i} \end{pmatrix} = \begin{pmatrix} 4.56 \\ -7.54 \\ -74.35 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta N}_{1s} \\ \bar{\Delta N}_{2s} \\ \bar{\Delta N}_{3s} \end{pmatrix} = \begin{pmatrix} 3.82 \\ -4.52 \\ -4.51 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta K}_{1i} \\ \bar{\Delta K}_{2i} \\ \bar{\Delta K}_{3i} \end{pmatrix} = \begin{pmatrix} 5.63 \\ -7.57 \\ -74.4 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta K}_{1s} \\ \bar{\Delta K}_{2s} \\ \bar{\Delta K}_{3s} \end{pmatrix} = \begin{pmatrix} 4.88 \\ -4.55 \\ -4.55 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta K}_{1m} \\ \bar{\Delta K}_{2m} \\ \bar{\Delta K}_{3m} \end{pmatrix} = \begin{pmatrix} 1.01 \\ 1.01 \\ 1.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta w}_1 \\ \bar{\Delta w}_2 \\ \bar{\Delta w}_3 \end{pmatrix} = \begin{pmatrix} 1.04 \\ -0.01 \\ -0.01 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta N}_{1m} \\ \bar{\Delta N}_{2m} \\ \bar{\Delta N}_{3m} \end{pmatrix} = \begin{pmatrix} -0.01 \\ 1.04 \\ 1.04 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta p}_{1L} \\ \bar{\Delta p}_{2L} \\ \bar{\Delta p}_{3L} \end{pmatrix} = \begin{pmatrix} 4.87 \\ -4.56 \\ -4.55 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta p}_{1s} \\ \bar{\Delta p}_{2s} \\ \bar{\Delta p}_{3s} \end{pmatrix} = \begin{pmatrix} 0.73 \\ 0 \\ 0 \end{pmatrix},$$

$$\begin{aligned}
 \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} &= \begin{pmatrix} 0.69 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} 4.9 \\ -4.53 \\ -4.52 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} 1.06 \\ 0.01 \\ 0.1 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} &= \begin{pmatrix} 0.33 \\ 0.01 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} 0.37 \\ -0.67 \\ -0.68 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} 1.06 \\ 0.01 \\ 0.01 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} &= \begin{pmatrix} 1.06 \\ 0.01 \\ 0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} -3.66 \\ 4.76 \\ 4.76 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} 1.06 \\ 0.01 \\ 0.01 \end{pmatrix}. \quad (23)
 \end{aligned}$$

A rise in region 3's amenity

We now study effects of the following improvement in region 3's amenity $\bar{\theta}_3 : 4.4 \Rightarrow 4.5$. The results are listed in (23). As the region 3's amenity is improved, people migrate to region 3 from the other two regions. As the other two regions' productivities are higher than region 3, the national income and national capital are reduced as more people move from higher productive regions to lower productive region. Region 3 expands the region's industrial and service sectors because of the improved living conditions. Region 3 shrinks the regional good sector because of the reduced demand from national demand. The other two regions have lower output levels of all the sectors. The wage rates are slightly changed. The land rent and price in region 3 rise and the variables in the other two regions fall. The household of region 3 has smaller lot size and the households of the other two regions have larger ones.

$$\begin{aligned}
 \bar{\Delta} Y &= -0.62, \quad \bar{\Delta} K = -0.73, \quad \bar{\Delta} r = -0.07, \\
 \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} &= \begin{pmatrix} -3.29 \\ -3.32 \\ 20.9 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -3.3 \\ -3.3 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -3.2 \\ -3.3 \\ 20.6 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} &= \begin{pmatrix} -3.8 \\ -4.9 \\ 292 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -3.3 \\ -3.4 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} -0.65 \\ -0.65 \\ -0.65 \end{pmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} &= \begin{pmatrix} -3.8 \\ -4.7 \\ 292 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -3.4 \\ -3.4 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} -0.67 \\ -0.67 \\ -0.67 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} &= \begin{pmatrix} -3.8 \\ -4.6 \\ 292 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -3.3 \\ -3.3 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} -0.61 \\ -0.61 \\ -0.61 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} &= \begin{pmatrix} 0.02 \\ 0.02 \\ 0.02 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} -3.2 \\ -3.3 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} -3.3 \\ -3.6 \\ 21 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -0.02 \\ -0.02 \\ -0.03 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} &= \begin{pmatrix} -0.02 \\ -0.02 \\ -0.02 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} -0.02 \\ -0.02 \\ -0.02 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} -0.02 \\ -0.02 \\ -0.03 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} &= \begin{pmatrix} -0.02 \\ -0.02 \\ -0.03 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} 3.4 \\ 3.5 \\ -17.4 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -0.02 \\ -0.02 \\ -0.03 \end{pmatrix}. \quad (24)
 \end{aligned}$$

The population having a stronger preference for 2's regional good

We now study effects of the following change in the population's preference for region 2's regional good $\gamma_{02} : 0.01 \Rightarrow 0.02$. The results are listed in (25). As the preference for a special region's good becomes stronger, the region expands the sector's scale and the sector employs more capital and labor. Region 3's other two sectors reduce their scales and employ less the two input factors. It is interesting to note that as the region's regional product is produced more, the region's population is reduced. More people migrate to region 1 which is more productive. As region 3's regional good is consumer good and the economic growth mechanism is neoclassical, a stronger desire for a consumer good tends to reduce wealth. The national income, regional incomes, capital stocks employed by each region, and national capital stock are reduced. The wage rates, land values

and land rents are all reduced. The prices of regional goods and services are slightly affected.

$$\begin{aligned}
 & \bar{\Delta}Y = -1.06, \quad \bar{\Delta}K = -3.44, \quad \bar{\Delta}r = 3.47, \\
 & \begin{pmatrix} \bar{\Delta}Y_1 \\ \bar{\Delta}Y_2 \\ \bar{\Delta}Y_3 \end{pmatrix} = \begin{pmatrix} -0.57 \\ -1.62 \\ -2.18 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}N_1 \\ \bar{\Delta}N_2 \\ \bar{\Delta}N_3 \end{pmatrix} = \begin{pmatrix} 0.48 \\ -0.32 \\ -1.15 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}K_1 \\ \bar{\Delta}K_2 \\ \bar{\Delta}K_3 \end{pmatrix} = \begin{pmatrix} -2.75 \\ -4.45 \\ -4.55 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}F_{1i} \\ \bar{\Delta}F_{2i} \\ \bar{\Delta}F_{3i} \end{pmatrix} = \begin{pmatrix} 6.07 \\ -41.5 \\ 116 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}F_{1s} \\ \bar{\Delta}F_{2s} \\ \bar{\Delta}F_{3s} \end{pmatrix} = \begin{pmatrix} -6.6 \\ -7.3 \\ -8.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}F_{1m} \\ \bar{\Delta}F_{2m} \\ \bar{\Delta}F_{3m} \end{pmatrix} = \begin{pmatrix} -7.1 \\ 86.1 \\ -7 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}N_{1i} \\ \bar{\Delta}N_{2i} \\ \bar{\Delta}N_{3i} \end{pmatrix} = \begin{pmatrix} 7.3 \\ -41 \\ 118 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}N_{1s} \\ \bar{\Delta}N_{2s} \\ \bar{\Delta}N_{3s} \end{pmatrix} = \begin{pmatrix} -5.6 \\ -6.4 \\ -7.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}N_{1m} \\ \bar{\Delta}N_{2m} \\ \bar{\Delta}N_{3m} \end{pmatrix} = \begin{pmatrix} -6 \\ 88 \\ -6.1 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}K_{1i} \\ \bar{\Delta}K_{2i} \\ \bar{\Delta}K_{3i} \end{pmatrix} = \begin{pmatrix} 3.6 \\ -42.8 \\ 111 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}K_{1s} \\ \bar{\Delta}K_{2s} \\ \bar{\Delta}K_{3s} \end{pmatrix} = \begin{pmatrix} -8.8 \\ -9.4 \\ -10 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}K_{1m} \\ \bar{\Delta}K_{2m} \\ \bar{\Delta}K_{3m} \end{pmatrix} = \begin{pmatrix} -9.1 \\ 82 \\ -9 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}w_1 \\ \bar{\Delta}w_2 \\ \bar{\Delta}w_3 \end{pmatrix} = \begin{pmatrix} -1.1 \\ -1 \\ -0.9 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}p_{1L} \\ \bar{\Delta}p_{2L} \\ \bar{\Delta}p_{3L} \end{pmatrix} = \begin{pmatrix} -9.8 \\ -10.4 \\ -11.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}p_{1s} \\ \bar{\Delta}p_{2s} \\ \bar{\Delta}p_{3s} \end{pmatrix} = \begin{pmatrix} -0.07 \\ 0.03 \\ 0.06 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}p_{1m} \\ \bar{\Delta}p_{2m} \\ \bar{\Delta}p_{3m} \end{pmatrix} = \begin{pmatrix} 0.07 \\ -0.03 \\ 0.03 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}R_1 \\ \bar{\Delta}R_2 \\ \bar{\Delta}R_3 \end{pmatrix} = \begin{pmatrix} -6.6 \\ -7.3 \\ -8 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}c_{1i} \\ \bar{\Delta}c_{2i} \\ \bar{\Delta}c_{3i} \end{pmatrix} = \begin{pmatrix} -7.1 \\ -7 \\ -6.9 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}c_{1s} \\ \bar{\Delta}c_{2s} \\ \bar{\Delta}c_{3s} \end{pmatrix} = \begin{pmatrix} -7 \\ -7 \\ -7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}c_{11} \\ \bar{\Delta}c_{21} \\ \bar{\Delta}c_{31} \end{pmatrix} = \begin{pmatrix} -7.1 \\ -7.1 \\ -7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}c_{12} \\ \bar{\Delta}c_{22} \\ \bar{\Delta}c_{32} \end{pmatrix} = \begin{pmatrix} 85.9 \\ 86.1 \\ 86.2 \end{pmatrix}, \\
 & \begin{pmatrix} \bar{\Delta}c_{13} \\ \bar{\Delta}c_{23} \\ \bar{\Delta}c_{33} \end{pmatrix} = \begin{pmatrix} -7.1 \\ -7 \\ -7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}l_1 \\ \bar{\Delta}l_2 \\ \bar{\Delta}l_3 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.3 \\ 1.16 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta}a_1 \\ \bar{\Delta}a_2 \\ \bar{\Delta}a_3 \end{pmatrix} = \begin{pmatrix} -7.1 \\ -7 \\ -6.9 \end{pmatrix}. \quad (25)
 \end{aligned}$$

The population having a stronger preference for lower residential density
 We now study effects of the following change in the population's preference for residential density $d : -0.06 \Rightarrow -0.1$. The population likes to choose area with low residential density. The results are listed in (26). As regions 2 and 3 have lower residential areas than region 1, the preference change makes the two regions more attractive. People migrate from region 1 to the other two regions. The redistribution of the population from the most productive region to the less productive regions reduces the national output and national capital stock. The economic structural changes occur as illustrated in (26). The wage rates, prices of services and regional products are slightly affected. The land value and rent in region 1 fall and the land values and land rents in the other two regions rise. Except the lot sizes, the other microeconomic variables are slightly affected.

$$\bar{\Delta} Y = -1.38, \quad \bar{\Delta} K = -1.59, \quad \bar{\Delta} r = -0.13,$$

$$\begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} -10.6 \\ 4.2 \\ 32.4 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -10.6 \\ 4.1 \\ 32.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -10.4 \\ 4.3 \\ 32 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -12.4 \\ 7.3 \\ 460 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -10.7 \\ 4.1 \\ 32.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} -1.45 \\ -1.45 \\ -1.45 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -12.4 \\ 7.2 \\ 460 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -10.7 \\ 4 \\ 32.5 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} -1.5 \\ -1.5 \\ -1.5 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} -12.3 \\ 7.3 \\ 460 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -10.6 \\ 4.2 \\ 32.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} -1.36 \\ -1.36 \\ -1.37 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.04 \\ 0.04 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} -10.5 \\ 4.2 \\ 32.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\begin{aligned}
 \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} -10.6 \\ 4.1 \\ 32.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -0.04 \\ -0.05 \\ -0.05 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} &= \begin{pmatrix} -0.05 \\ -0.04 \\ -0.05 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} -0.04 \\ -0.04 \\ -0.05 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} -0.04 \\ -0.05 \\ -0.05 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} &= \begin{pmatrix} -0.04 \\ -0.05 \\ -0.05 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} 11.9 \\ -4 \\ -24.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -0.04 \\ -0.05 \\ -0.05 \end{pmatrix}. \quad (26)
 \end{aligned}$$

The productivity of region 3's service sector being enhanced

Another question is what will happen if a region's service sector improves its productivity. The results are listed in (27). As services cannot be consumed by residents in any other region, a higher productivity in services tends to make the region more attractive as the service price becomes lower. We now examine effects of the following change in productivity of region 3's service sector $A_{3s} : 0.8 \Rightarrow 0.9$. The productivity change makes region 3 more attractive. People migrate from regions 1 and 2 to region 3. The redistribution of the population from the most productive regions to the least productive region reduces the national output and national capital stock. The rate of interest falls. The economic structural changes occur as illustrated in (27). The wage rates are slightly affected. Region 3's price of service falls and the prices of services in the other two regions are slightly affected. The prices of regional products are slightly affected. The land value and rent in region 3 rise and the land values and land rents in the other two regions fall. Except the lot sizes and the consumption level of region 3's services, the other microeconomic variables are slightly affected.

$$\begin{aligned}
 \bar{\Delta} Y &= -0.22, \quad \bar{\Delta} K = -0.25, \quad \bar{\Delta} r = -0.02, \\
 \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} &= \begin{pmatrix} -1.15 \\ -1.16 \\ 7.28 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -1.16 \\ -1.16 \\ 7.33 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -1.12 \\ -1.15 \\ 7.16 \end{pmatrix},
 \end{aligned}$$

$$\begin{aligned}
 \begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} &= \begin{pmatrix} -1.33 \\ -1.64 \\ 102 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -1.17 \\ -1.17 \\ 20.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} -0.23 \\ -0.23 \\ -0.23 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} &= \begin{pmatrix} -1.33 \\ -1.64 \\ 102 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -1.17 \\ -1.18 \\ 7.3 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} -0.23 \\ -0.23 \\ -0.23 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} &= \begin{pmatrix} -1.31 \\ -1.62 \\ 102 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -1.15 \\ -1.16 \\ 7.33 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} -0.2 \\ -0.2 \\ -0.2 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} &= \begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} -1.14 \\ -1.15 \\ 7.34 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -11.1 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} &= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} -1.2 \\ -1.2 \\ 7.3 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} &= \begin{pmatrix} -0.01 \\ -0.01 \\ 12.5 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} &= \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1.2 \\ -6.8 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}. \quad (27)
 \end{aligned}$$

The population's propensity to save being enhanced

Although issues related to saving rates have been examined in different analytical frameworks, there are only a few studies on issues related to special agglomeration. We now deal with impact of the population's propensity to save on regional distributions of capital, people and outputs. We allow the propensity to save to be changed as follows: $\lambda_0 : 0.7 \Rightarrow 0.71$. The results are listed in (28). The enhanced propensity to save enhances the national income and national capital. The rate of interest

falls. Region 1 becomes more attractive. Some people migrate to region 1 from the other two regions, which also implies a reduction in region 1's lot size and rises in the other two regions' lot sizes. The land values in regions 1 and 2 are increased and the land value in region 3 is reduced. The land rents in 1 is increased and the land rents in regions 2 and 3 are reduced. It is important to observe the opposite effects on the land rent and value. Region 1's total income rises and the other two regions' incomes fall. All the regional good sectors expand the production scales. Except the lot sizes, the representative households in all the regions have more wealth, consume more goods and services.

$$\bar{\Delta} Y = 0.62, \quad \bar{\Delta} K = 1.73, \quad \bar{\Delta} r = -1.55,$$

$$\begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} 1.54 \\ -0.24 \\ -1.99 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} 1.03 \\ -0.71 \\ -2.43 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} 2.65 \\ 0.81 \\ -0.94 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} 1.86 \\ -0.5 \\ -32 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} 1.39 \\ -0.34 \\ -2.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} 0.53 \\ 0.48 \\ 0.51 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} 1.34 \\ 0.97 \\ -32.5 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} 0.9 \\ -0.8 \\ -2.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} -0.01 \\ 0.03 \\ 0.06 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} 2.98 \\ 0.55 \\ -31.5 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} 2.53 \\ 0.7 \\ -1.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} 1.6 \\ 1.6 \\ 1.5 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} = \begin{pmatrix} -0.03 \\ 0.02 \\ -0.02 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0.51 \\ 0.47 \\ 0.44 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} 3.06 \\ 1.25 \\ -0.54 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} = \begin{pmatrix} 0.03 \\ -0.02 \\ -0.03 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} 1.42 \\ -0.35 \\ -2.12 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.36 \\ 0.32 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} 0.36 \\ 0.37 \\ 0.35 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} 0.43 \\ 0.39 \\ 0.35 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} 0.38 \\ 0.34 \\ 0.31 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} = \begin{pmatrix} 0.41 \\ 0.37 \\ 0.34 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} -1.01 \\ 0.71 \\ 2.49 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} 1.83 \\ 1.79 \\ 1.76 \end{pmatrix}. \quad (28)$$

The population's propensity to consume housing being enhanced

We allow the propensity to consume housing be changed as follows: $\eta_0 : 0.04 \Rightarrow 0.05$. The results are listed in (29). The population migrates to areas with low residential densities. As regions 2 and 3 have lower residential areas than region 1, the preference change makes the two regions more attractive. People migrate from region 1 to regions 2 and 3. The lot sizes are correspondingly changed. The land rents in all the regions are enhanced. The land value of region 1 is reduced and the land values of the two regions are increased. The national income and capital stock are reduced. The rate of interest also falls. Region 1's income falls and the other two regions' incomes rise. Except the changes in the lot sizes, the representative households of all the regions have less wealth and consume more goods and services.

$$\bar{\Delta} Y = -4.98, \bar{\Delta} K = -13.7, \bar{\Delta} r = 14.8,$$

$$\begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} = \begin{pmatrix} -10.6 \\ 0.56 \\ 10 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -6.5 \\ 4.9 \\ 14.5 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -18.8 \\ -8.4 \\ 0.28 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -12.8 \\ 1.9 \\ 176 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -9.4 \\ 1.4 \\ 10.9 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} -4.3 \\ -3.9 \\ -4.2 \end{pmatrix},$$

$$\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -8.7 \\ 6.3 \\ 187 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -5.5 \\ 5.9 \\ 15.6 \end{pmatrix}, \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} 0.42 \\ 0.08 \\ -0.23 \end{pmatrix},$$

$$\begin{aligned}
 \begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} &= \begin{pmatrix} -20.8 \\ -7.2 \\ 152 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -18 \\ -7.5 \\ 1.6 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} -12.8 \\ -12.6 \\ -12.3 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} &= \begin{pmatrix} 0.28 \\ -0.14 \\ 0.13 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} -4.4 \\ -4.1 \\ -3.8 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} -1.7 \\ 10.6 \\ 21.1 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} &= \begin{pmatrix} -0.37 \\ 0.14 \\ 0.26 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} 13 \\ 27 \\ 39 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -3.85 \\ -3.2 \\ -2.6 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} &= \begin{pmatrix} -3.2 \\ -3.3 \\ -3.1 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} -3.75 \\ -3.42 \\ -3.12 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} -3.35 \\ -3 \\ -2.7 \end{pmatrix}, \\
 \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} &= \begin{pmatrix} -3.6 \\ -3.28 \\ -2.98 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} 6.9 \\ -4.6 \\ -12.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -3.5 \\ -3.2 \\ -2.9 \end{pmatrix}. \quad (29)
 \end{aligned}$$

Region 3's land available for residential use being enlarged

We now allow region 3's land available for residential use to be enlarged as follows: $L_3 : 7 \Rightarrow 8$. The results are listed in (30). People migrate from regions 1 and 2 to region 3. The lot sizes in all the regions are increased. The land values and rents in all the regions are reduced. The national income and capital stock are reduced. The rate of interest also falls. Region 1's and region 2's incomes fall and region 3's income rises. The representative households of all the regions have less wealth and consume more goods and services.

$$\begin{aligned}
 \bar{\Delta} Y &= -0.14, \quad \bar{\Delta} K = -0.16, \quad \bar{\Delta} r = -0.02, \\
 \begin{pmatrix} \bar{\Delta} Y_1 \\ \bar{\Delta} Y_2 \\ \bar{\Delta} Y_3 \end{pmatrix} &= \begin{pmatrix} -0.74 \\ -0.74 \\ 4.66 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_1 \\ \bar{\Delta} N_2 \\ \bar{\Delta} N_3 \end{pmatrix} = \begin{pmatrix} -0.74 \\ -0.75 \\ 4.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_1 \\ \bar{\Delta} K_2 \\ \bar{\Delta} K_3 \end{pmatrix} = \begin{pmatrix} -0.72 \\ -0.74 \\ 4.6 \end{pmatrix},
 \end{aligned}$$

$$\begin{aligned}
 &\begin{pmatrix} \bar{\Delta} F_{1i} \\ \bar{\Delta} F_{2i} \\ \bar{\Delta} F_{3i} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -1.05 \\ 65.3 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1s} \\ \bar{\Delta} F_{2s} \\ \bar{\Delta} F_{3s} \end{pmatrix} = \begin{pmatrix} -0.75 \\ -0.75 \\ 4.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} F_{1m} \\ \bar{\Delta} F_{2m} \\ \bar{\Delta} F_{3m} \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.15 \\ -0.15 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} N_{1i} \\ \bar{\Delta} N_{2i} \\ \bar{\Delta} N_{3i} \end{pmatrix} = \begin{pmatrix} -0.85 \\ -1.05 \\ 65.3 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1s} \\ \bar{\Delta} N_{2s} \\ \bar{\Delta} N_{3s} \end{pmatrix} = \begin{pmatrix} -0.75 \\ -0.76 \\ 4.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} N_{1m} \\ \bar{\Delta} N_{2m} \\ \bar{\Delta} N_{3m} \end{pmatrix} = \begin{pmatrix} -0.15 \\ -0.15 \\ -0.15 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} K_{1i} \\ \bar{\Delta} K_{2i} \\ \bar{\Delta} K_{3i} \end{pmatrix} = \begin{pmatrix} -0.84 \\ -1.04 \\ 65.3 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1s} \\ \bar{\Delta} K_{2s} \\ \bar{\Delta} K_{3s} \end{pmatrix} = \begin{pmatrix} -0.74 \\ -0.74 \\ 4.7 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} K_{1m} \\ \bar{\Delta} K_{2m} \\ \bar{\Delta} K_{3m} \end{pmatrix} = \begin{pmatrix} -0.14 \\ -0.14 \\ -0.14 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} w_1 \\ \bar{\Delta} w_2 \\ \bar{\Delta} w_3 \end{pmatrix} = \begin{pmatrix} 0.01 \\ 0.01 \\ 0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1L} \\ \bar{\Delta} p_{2L} \\ \bar{\Delta} p_{3L} \end{pmatrix} = \begin{pmatrix} -0.73 \\ -0.74 \\ -8.38 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} p_{1s} \\ \bar{\Delta} p_{2s} \\ \bar{\Delta} p_{3s} \end{pmatrix} = \begin{pmatrix} \bar{\Delta} p_{1m} \\ \bar{\Delta} p_{2m} \\ \bar{\Delta} p_{3m} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} R_1 \\ \bar{\Delta} R_2 \\ \bar{\Delta} R_3 \end{pmatrix} = \begin{pmatrix} -0.75 \\ -0.75 \\ -8.4 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1i} \\ \bar{\Delta} c_{2i} \\ \bar{\Delta} c_{3i} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{1s} \\ \bar{\Delta} c_{2s} \\ \bar{\Delta} c_{3s} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} c_{11} \\ \bar{\Delta} c_{21} \\ \bar{\Delta} c_{31} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{12} \\ \bar{\Delta} c_{22} \\ \bar{\Delta} c_{32} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} c_{13} \\ \bar{\Delta} c_{23} \\ \bar{\Delta} c_{33} \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}, \\
 &\begin{pmatrix} \bar{\Delta} l_1 \\ \bar{\Delta} l_2 \\ \bar{\Delta} l_3 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.8 \\ 9.2 \end{pmatrix}, \quad \begin{pmatrix} \bar{\Delta} a_1 \\ \bar{\Delta} a_2 \\ \bar{\Delta} a_3 \end{pmatrix} = \begin{pmatrix} -0.01 \\ -0.01 \\ -0.01 \end{pmatrix}.
 \end{aligned} \tag{30}$$

Concluding Remarks

This paper built a general equilibrium interregional trade model with any number of regional economies by unifying the Solow-Uzawa growth and H-O trade theories. The model made a synthesis of the Solow-Uzawa growth and H-O trade theories with Zhang's utility function for describing behavior of households. We study the roles of population, land, preferences and technological differences between regions in determining dynamics of national wealth, patterns of interregional trade, economic

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structures, income and wealth distributions between different regional households in a general equilibrium dynamic framework. The paper dealt with endogenous wealth accumulation, land values, residential land-use, and labor and capital distribution between sectors and between regions under perfectly competitive markets and free trade. Each region has three sectors, producing one national homogenous tradable capital good, specifying in producing one tradable commodity, and supplying non-tradable goods and services. A new aspect of the study is to determine land value and rent and how these variables are related to growth and inequality. The study made simulation of the model. We examined the effects on the equilibrium point in exogenous changes of the land, the population, a region's amenity, and the preferences for a regional good, saving, housing, and residential density. In our model the economic structures and interactions between different determinants of regional economic growth are delicately interrelated. We might get more insights from further simulation. Our comparative analysis was limited to a few cases. The Solow model, the Uzawa two-sector growth, the H-O model, and the Oniki-Uzawa trade model are developed and generalized in different directions. Many limitations of our model become apparent in the light of the sophistication of the literature based on these models. It is straightforward for us to make analysis of behavior of the model by applying different forms of production and utility functions. Also issues related to tax competition between regions have caused great attention in economic geography. We may generalize and extend our model on the basis of the traditional literature in the neoclassical growth model and trade. We may introduce public sectors and taxation to examine spatial agglomeration.

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Appendix: Proving the Lemma

Equations (2) imply

$$z_j \equiv \frac{r + \delta_{jk}}{w_j} = \frac{\bar{\alpha}_{ji} N_{ji}}{K_{ji}} = \frac{\bar{\alpha}_{jm} N_{jm}}{K_{jm}} = \frac{\bar{\alpha}_{js} N_{js}}{K_{js}}, \quad (\text{A1})$$

where $\bar{\alpha}_{jq} \equiv \alpha_{jq} / \beta_{jq}$. From (2) and (A1), we have

$$r(z_j) = \frac{\alpha_{ji} A_{ji} z_j^{\beta_{ji}}}{\bar{\alpha}_{ji}^{\beta_{ji}}} - \delta_{jk}, \quad w_j(z_j) = \frac{r + \delta_{jk}}{z_j}. \quad (\text{A2})$$

From (A2) we have

$$z_j(z_1) = \left(\frac{r(z_1) + \delta_{jk}}{\alpha_{ji} A_{ji}} \right)^{1/\beta_{ji}} \bar{\alpha}_{ji}. \quad (\text{A3})$$

From (3) and (A1)

$$p_{jq}(z_1) = \frac{w_j z_j^{\alpha_{jq}}}{\beta_{jq} A_{jq} \bar{\alpha}_{jq}^{\alpha_{jq}}}. \quad (\text{A4})$$

Equations (5) and (6) imply

$$\hat{y}_j = r \bar{k}_j + w_j + \sum_{n=1}^J R_n \bar{l}_{jn} + a_j. \quad (\text{A5})$$

Insert (3) in (A5)

$$\hat{y}_j = r \bar{k}_j + w_j + r \sum_{n=1}^J p_{Ln} \bar{l}_{jn} + a_j. \quad (\text{A6})$$

The definition of a_j implies

$$\hat{y}_j(z_1, a_j) = (1 + r)a_j + w_j. \quad (\text{A7})$$

Equations (13) and (A1) imply

$$\bar{\alpha}_{ji} N_{ji} + \bar{\alpha}_{jm} N_{jm} + \bar{\alpha}_{js} N_{js} = z_j K_j. \quad (\text{A8})$$

Equations (10) and (15) imply

$$\chi \hat{y}_j N_j = p_{js} F_{js}. \quad (\text{A9})$$

Substitute (2) into (A7)

$$\chi \hat{y}_j N_j = \frac{w_j N_{js}}{\beta_{js}}. \quad (\text{A10})$$

Insert (A5) in (A10)

$$N_{js} = (\omega_j a_j + \chi \beta_{js}) N_j, \quad (\text{A11})$$

where $\omega_j \equiv ((1+r)/w_j) \chi \beta_{js}$. Insert (A11) in (14)

$$N_{ji} = (1 - \omega_j a_j - \chi \beta_{js}) N_j - N_{jm}. \quad (\text{A12})$$

Insert (10) in (16)

$$\gamma_j \sum_{n=1}^J \hat{y}_n N_n = p_{jm} F_{jm}. \quad (\text{A13})$$

Insert (2) and (A7) in (A13)

$$N_{jm} = \tilde{\gamma}_j \sum_{n=1}^J N_n a_n + \sum_{n=1}^J \bar{\gamma}_{jn} N_n, \quad (\text{A14})$$

where

$$\tilde{\gamma}_j \equiv \frac{\beta_{jm} \gamma_j (1+r)}{w_j}, \quad \bar{\gamma}_{jn} \equiv \frac{\beta_{jm} \gamma_j w_n}{w_j}.$$

Insert (14) in (A8)

$$\bar{\alpha}_{ji} N_j + \tilde{\alpha}_{jm} N_{jm} + \tilde{\alpha}_{js} N_{js} = z_j K_j, \quad (\text{A15})$$

where

$$\tilde{\alpha}_{jm} \equiv \bar{\alpha}_{jm} - \bar{\alpha}_{ji}, \quad \tilde{\alpha}_{js} \equiv \bar{\alpha}_{js} - \bar{\alpha}_{ji}.$$

Substituting (A11) and (A12) into (A15)

$$\sum_{n=1}^J \tilde{\omega}_{jn} N_n a_n + \sum_{n=1}^J \bar{\omega}_{jn} N_n = K_j, \quad (\text{A16})$$

where

$$\bar{\omega}_{jj} \equiv \frac{\bar{\alpha}_{ji} + \tilde{\alpha}_{js} \chi \beta_{js}}{z_j} + \frac{\tilde{\alpha}_{jm} \bar{\gamma}_{jj}}{z_j}, \quad \bar{\omega}_{jn} \equiv \frac{\tilde{\alpha}_{jm} \bar{\gamma}_{jn}}{z_j}, \quad n \neq j$$

$$\tilde{\omega}_{jj} \equiv \frac{\tilde{\alpha}_{js} \omega_j}{z_j} + \frac{\tilde{\alpha}_{jm} \tilde{\gamma}_j}{z_j}, \quad \tilde{\omega}_{jn} \equiv \frac{\tilde{\alpha}_{jm} \tilde{\gamma}_j}{z_j}, \quad n \neq j.$$

From (18) and (A15) we have

$$K = \sum_{n=1}^J \tilde{\omega}_{jn} N_n a_n + \sum_{n=1}^J \bar{\omega}_{jn} N_n, \quad (\text{A17})$$

where

$$\tilde{\omega}_n \equiv \sum_{j=1}^J \tilde{\omega}_{jn}, \quad \bar{\omega}_n \equiv \sum_{j=1}^J \bar{\omega}_{jn}.$$

Substitute $l_j = L_j / N_j$, (9) and (10) into (9)

$$U_j = \tilde{\theta}_j N_j^{d-\eta_0} \hat{y}_j^b, \quad (\text{A18})$$

where

$$\tilde{\theta}_j(z_1, (a_j)) \equiv \bar{\theta}_j L_j^{\eta_0} \xi^{\xi_0} \left(\frac{\chi}{p_{js}} \right)^{\chi_0} \chi^{\lambda_0} \prod_{n=1}^J \left(\frac{\gamma_j}{p_{nm}} \right)^{\gamma_{0n}},$$

$$b \equiv \xi_0 + \chi_0 + \lambda_0 + \sum_{n=1}^J \gamma_{0n}.$$

Apply $U_j = U_1$ to (A18)

$$N_j = \Psi_j N_1, \quad (\text{A19})$$

where

$$\Psi_j(z_1, (a_j)) \equiv \left(\frac{\tilde{\theta}_1 \hat{y}_1^b}{\tilde{\theta}_j \hat{y}_j^b} \right)^{\frac{1}{(d-\eta_0)}}.$$

Insert (A10) in (13)

$$N_1(z_1, (a_j)) = \frac{N}{\sum_{j=1}^J \Psi_j}, \quad \Psi_1 = 1. \quad (\text{A20})$$

It is easy to confirm that we can express all the variables as functions of z_1 and (a_j) as follows: r by (A2) $\rightarrow w_j$ by (A2) $\rightarrow z_j$ by (A3) $\rightarrow p_{jq}$ by (A4) $\rightarrow \hat{y}_j$ by (A7) $\rightarrow c_{ji}, c_{js}, c_{jm}$, and s_j by (7) $\rightarrow N_1$ by (A20) $\rightarrow N_j$ by (A19) $\rightarrow K$ by (A16) $\rightarrow K_j$ by (A15) $\rightarrow N_{jm}$ by (A14) $\rightarrow N_{ji}$ by (A12) $\rightarrow N_{js}$ by (A11) $\rightarrow K_{jq}$ by (A1) $\rightarrow F_{jq}$ by (2) $\rightarrow l_j$ by (20) $\rightarrow R_j$ by (7). Applying this procedure, (11) and (17), we get

$$\dot{K} = \Lambda_0(z_1, (a_j)) \equiv \sum_{j=1}^J F_{ji} - \sum_{j=1}^J \delta_{kj} K_j - \sum_{j=1}^J c_{ji} N_j, \quad (\text{A21})$$

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$$\dot{a}_j = \Omega_j(z_1, (a_j)) \equiv s_j - a_j. \quad (\text{A22})$$

Here, we don't provide explicit expressions of the functions as they are tedious. We represent (A16) as

$$K = \Lambda(z_1, (a_j)) \equiv \sum_{n=1}^J \tilde{\omega}_{jn} N_n a_n + \sum_{n=1}^J \bar{\omega}_{jn} N_n. \quad (\text{A23})$$

We take derivatives of (A23) with respect to t to get

$$\dot{K} = \frac{\partial \Lambda}{\partial z_1} \dot{z}_1 + \sum_{j=1}^J \frac{\partial \Lambda}{\partial a_j} \dot{a}_j. \quad (\text{A24})$$

Insert (A22) in (A24)

$$\dot{K} = \frac{\partial \Lambda}{\partial z_1} \dot{z}_1 + \sum_{j=1}^J \Omega_j \frac{\partial \Lambda}{\partial a_j}. \quad (\text{A25})$$

where (A19) is applied. Equal (A25) and (A21)

$$\dot{z}_1 = \bar{\Lambda}(z_1, (a_j)) \equiv \left(\Lambda_0 - \sum_{j=1}^J \Omega_j \frac{\partial \Lambda}{\partial a_j} \right) \left(\frac{\partial \Lambda}{\partial z_1} \right)^{-1}. \quad (\text{A26})$$

We thus proved the lemma.